Control of Single-Phase Four-Quadrant PWM Rectifier for Traction Systems

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Abstract — This research has been motivated by industrial demand for a single phase four-quadrant rectifier for traction systems. This paper presents an advanced control structure design for a single phase PWM rectifier. The control structure consists of a proportional-resonant controller using a fast phase angle and frequency estimator. The estimation algorithm is derived from the weighted least-squares estimation method. The feasibility of the proposed control structure is confirmed by experimental tests performed on designed laboratory prototype.

Keywords — rectifier, converter control, PWM, resonant control, phase angle estimation, traction application

I. INTRODUCTION

At the beginning of the 1970’s, the research and development of electric traction schemes of an electric locomotive with the traction voltage 15kV/16,7Hz (Germany, Switzerland, Austria, Sweden, Norway) and 25kV/50Hz (France, Spain, etc.) has been oriented almost exclusively to implementation of a three-phase induction motor as an electric traction motor which had replace a contact DC traction motor with series and separate excitation. Gradually, the traction AC single-phase voltage has been reduced by a traction vehicle transformer with voltage transfer which allows to generate a DC-link voltage for traction frequency converters to supply the three-phase induction traction motor (ITM) with nominal line-to-line voltage (approx. 2-2.2kV). The marginal power of ITM directed to hundreds kW, today the top locomotives has power about 1.2-1.6MW. Use this power with four or six ITM in one vehicle requires that from traction line was consumed power about 1.2-1.6 MW.

In a single phase system, the reference of the line current is a 50 Hz AC signal under the steady state, and the current controller is required to track such an AC reference. There have been several control schemes for an AC tracking controller such as the hysteresis control methods, the predictive control methods, and so on [2], [14]. In this work is used the resonant control approach [4], [7-10] for an AC tracking controller. The proposed control scheme using the PR (Proportional-Resonant) controller that is capable of tracking a sinusoidal line current reference without an additional prediction or an extremely high control gain.

When electrical railway traction crosses over to the power section supplied by another voltage source, the amplitude and/or the phase angle of a source voltage may change in a step manner. In this case, the normal phase angle detector, such as a phase locked loop (PLL) and a lowpass (or notch) filter, typically generates a phase delay, results in a sluggish response, and causes some time critical machine to malfunction. In this work, a source voltage estimator is used based on the weighted least squares estimation (WLSE) method [9]. The proposed estimator provides the phase angle, the frequency, and the magnitude information of the source voltage’s fundamental component without a delay.

PWM rectifier

The rectifier controlled by pulse width modulation (PWM) consumes current of required shape, which is mostly sinusoidal. It works with a given phase displacement between the consumed current and the supply voltage. The power factor can also be controlled and there are minimal effects on the supply network.

The main features of PWM rectifiers are [5]:

- bi-directional power flow,
- nearly sinusoidal input current,
- regulation of input power factor to unity,
- low harmonic distortion of line current (THD below 5%),
- adjustment and stabilization of DC link voltage (or current),
- reduced capacitor (or inductor) size due to the continuous current,
- properly operated under line voltage distortion and line frequency variations.
PWM rectifiers can be divided into two groups according to power circuit connection [1]:

- **voltage rectifiers** (called boost rectifier – increases voltage): requires higher voltage on the DC side than the maximum value of the supply voltage. The rectified voltage on the output is smoother than the output voltage of the current type rectifier. They also require a more powerful microprocessor for their control. Output voltage lower than the voltage on input side can be obtained only with increased reactive power consumption.

- **current rectifiers** (called buck rectifier – decreases voltage): the maximum value of the supply voltage must be higher than the value of the rectified voltage. The main advantage is that the rectified voltage is regulated from zero. They are suitable for work with DC loads (DC motors, current inverters).

The PWM single phase rectifier consists of 4 IGBTs connected in full bridge [12] is shown in Fig. 1.

The source power is supplied through a transformer Tr and the input inductance L. The output DC link voltage $U_{DC}$ is filtered by capacitor C and fed into a 3-phase inverter that drives the traction. Supplied voltage $U_S$ and the voltage at the rectifier input $U_R$ are sinusoidal waveforms separated by the input inductance. Therefore the energy flow depends on the angle between these two phases. See the phase diagram in Fig. 2a [1], [5], [11].

The power transferred from the supply to the input terminals of the rectifier is:

$$P = \frac{U_S U_R \sin \delta}{X_S} = U_S I_S \cos \phi$$  \hfill (1)

where $U_S$ is RMS value of input supply voltage (V), $U_R$ RMS value of first harmonics consumed by AC rectifier input (V), $\delta$ phase displacement between phasors $U_S$ a $U_R$ (deg), $X_S$ input inductor reactance at 50Hz ($\Omega$), $\phi$ power factor.

In order to make the rectified voltage constant the input and output powers must be balanced. Then as the phasor diagram in Fig. 2a shows:

$$I_S \cos \phi = \frac{U_R \sin \delta}{X_S},$$  \hfill (2)

$$I_S \sin \phi = \frac{U_S - U_R \cos \delta}{X_S}. \hfill (3)$$

As long as the reactive power consumed is equal to zero the power factor is equal to unity. Therefore (2) and (3) can be adapted to:

$$I_S X_S = U_R \sin \delta,$$  \hfill (4)

$$U_S = U_R \cos \delta.$$  \hfill (5)

Phasor diagrams of the rectifier which works both as a rectifier and as an inverter are shown in Figs. 2b and 2c.

The aim is to control the rectifier in such a way that it consumes harmonical current from the supply network which is in phase with the supply voltage. This can be achieved by controlling the rectifier by pulse width modulation.

II. PROPOSED CONTROL AND ESTIMATION ALGORITHM

A. Phase angle and frequency estimation

The phase angle of a source voltage is used to calculate and control the flow of active/reactive power. The phase angle is a critical piece of information for the operation of most power conditioning equipment, such as pulse width modulation AC/DC converter, uninterruptible power supplies (UPS), AC voltage compensators, static VAR compensators (SVC), active harmonic filters etc. In power conditioning equipment, the exact value of a positive sequence is needed to achieve the unity power factor and constant output voltage, whereas the exact value of a negative sequence is needed for unbalance compensation.

A fast phase angle and frequency estimator is presented, which is capable of estimating the phase angle and the frequency of the source voltage even under a highly distorted source voltage condition or sudden amplitude, phase angle, or
frequency changing condition. The algorithm is derived from the weighted least squares estimation (WLSE) [8-10].

A single phase voltage \( U_{S} \) is expressed such that

\[
U_{S}(t) = U_{m} \sin(\omega t + \varphi) = U_{d} \cos(\omega t) + U_{q} \sin(\omega t),
\]

where \( U_{m} \) is amplitude of source voltage (V), \( \omega \) constant angular frequency (rad/s), \( \varphi \) phase angle (deg),

\[
U_{d} = U_{m} \cos(\varphi), \quad U_{q} = U_{m} \sin(\varphi).
\]

By applying the WLSE method to (6), the estimation \( \hat{U}_{d} \) and \( \hat{U}_{q} \) are obtained from \( U_{S} \) such that

\[
\dot{x}(t_i) = R(t_i)\dot{U}_{S}(t_i) - \lambda H(t_i)\dot{x}(t_{i-1}), \quad i = 1, 2, 3, \ldots
\]

where

\[
\dot{x}(t_i) = \begin{bmatrix} U_d(t_i) & U_q(t_i) \end{bmatrix}^T,
\]

\[
H(t_i) = \begin{bmatrix} \sin(\alpha t_i) & \cos(\alpha t_i) \end{bmatrix},
\]

\[
R(t_i) = P(t_i)H(t_i)^T \left( I + H(t_i)P(t_i)H(t_i)^T \right)^{-1},
\]

\[
P(t_i) = \lambda^2 \left( P(t_{i-1}) - R(t_i)H(t_i)P(t_i)H(t_i)^T \right),
\]

\( \lambda \in (0, 1) \) is the forgetting factor, initial conditions:

\[
\dot{x}(t_0) = 0, \quad P(t_0) = \gamma I \in \mathbb{R}^{2 \times 2},
\]

\( \gamma > 0 \) is the initial covariance constant.

The noise immunity of the WLSE estimator can be increased by selecting a larger forgetting factor \( \lambda \) and faster convergence can be achieved by choosing the larger \( \gamma \).

The phase angles estimation is obtained from \( \hat{U}_{d} \) and \( \hat{U}_{q} \) such that

\[
\dot{\phi}(t_i) = \arctan \left( \frac{2(\hat{U}_{d}(t_i)\hat{U}_{q}(t_i))}{1 + \hat{U}_{d}(t_i)^2 + \hat{U}_{q}(t_i)^2} \right), \quad i = 1, 2, 3, \ldots
\]

where \( \arctan2 \) is the arc-tangent function.

The frequency information is quite important for phase angle estimation algorithm. The phase angle estimation algorithm can be extended to the estimation of \( \omega \) when the frequency varies. When the frequency estimate \( \hat{\omega} \) is not equal to the real frequency \( \omega \), the estimated phase angle \( \varphi \) varies such that

\[
\Delta \hat{\phi} = \hat{\phi}(t_i) - \hat{\phi}(t_{i-1}) = (\omega - \hat{\omega})(t_i - t_{i-1}), \quad i = 1, 2, 3, \ldots
\]

We can recognize that if \( \Delta \hat{\phi} \neq 0 \), then there is a frequency estimation error. The basic idea for updating \( \hat{\omega} \) is to employ a PI controller (10) so that \( \Delta \hat{\phi} \) is nullified

\[
\hat{\omega}(t_i) = \hat{\omega}(t_0) + K_{P\varphi} e(t_i) + K_{I\varphi} \sum_{j=1}^{i} e(t_j), \quad i = 1, 2, 3, \ldots
\]

where \( e(t_i) = \hat{\phi}(t_i) - \hat{\phi}(t_{i-1}) \) is error,

\( K_{P\varphi} \) proportional gain,

\( K_{I\varphi} \) integral gain.

The estimated source voltage \( \hat{U}_{S} \) is then obtained as

\[
\hat{U}_{S}(t_i) = \hat{U}_{d}(t_i)\sin(\hat{\theta}(t_i)) + \hat{U}_{q}(t_i)\cos(\hat{\theta}(t_i)), \quad i = 1, 2, 3, \ldots
\]

B. Voltage and current controller

The proposed control system consists of the DC-link voltage controller, the current controller, the phase angle estimator and the PWM generator [7], [9], see the structure in Fig. 3.

The DC-link voltage controller is implemented by using a conventional proportional-integral (PI) controller whose output is the amplitude of the current reference \( I_{m} \) and transfer function is given by

\[
G_{PI}(s) = K_{Pu} + \frac{K_{Iu}}{s}.
\]

where \( K_{Pu} \) and \( K_{Iu} \) are the proportional and the integral control gain, respectively.

The current reference \( I_{r} \), is constructed by multiplying the synchronized signal with the source voltage: \( I_{r} = I_{m} \sin(\hat{\theta} + \varphi) \). The sinusoidal current reference \( I_{r} \), is fed into the current controller.

The current controller is constructed based on the proportional-resonant (PR) controller whose transfer function [4] is given by

\[
G_{PR}(s) = K_{Pi} + \frac{2K_{Ri} s}{s^2 + \omega^2},
\]

where \( K_{Pi} \) and \( K_{Ri} \) are the proportional and the resonant control gain, respectively,

\( \omega \) is fundamental angular frequency of the source current (rad/s).

Figure 3. Control structure of a single phase PWM rectifier
The time domain response of the resonant controller when $K_P = 0.5$, $K_I = 2$, $f = 50Hz$ with sinusoidal input $\sin(\omega t)$ is shown in Fig. 4.

It can be seen from Fig. 4 that the gain of the transfer function (13) is infinity at $\omega$ the output is in phase with the input signal, but the amplitude is amplified with time. With the resonant control method, one can track the high frequency sinusoidal current reference without increasing the switching frequency nor adopting an extremely large control gain.

For proper operation of PWM rectifier a minimum DC-link voltage is required [1], [3], [5]. Generally, it can be determined by the maximum value of the supply voltage $U_S$ that is

$$U_{DC\min} > U_{S\max} \quad \text{or} \quad U_{DC\min} > \sqrt{2} U_{S(RMS)}.$$  

If this condition is not fulfilled, the full control of the input current is not possible. Defining the natural DC-link voltage value (as it is possible to obtain in case of not operating transistors) the freewheeling diodes constitute a standard diode bridge. Typically, the reference value for the controlled DC-link voltage should be chosen about 10% above the natural DC-link voltage. The unity power factor required for PWM rectifier operation can be obtained in case of

$$\frac{U^2}{R} = \frac{U^2}{S} + \frac{U^2}{L}.$$  

The voltage drop across the inductor $U_L$ depends on reactance $L$ of the inductor at the input frequency and on the input current $I_S$. The magnitude of the switching voltage vectors depends on the DC-link voltage level. The inductor has to be designed carefully because low inductance will give a high current ripple and will make the design more depending on the line impedance. The high value of inductance will give a low current ripple, but simultaneously reduces the operation range of the rectifier. A high current (high power) through the inductance requires either a high DC-link voltage or a low inductance (low impedance) [5].

III. EXPERIMENTAL RESULTS

The control structure of a single phase PWM rectifier has been confirmed by experimental tests performed on designed laboratory prototype. Block diagram of the PWM rectifier control system is shown in Fig. 5. The power part is realized by four IGBT transistors and the digital signal processor TMS320F28335 (Texas Instruments) was chosen as controller and for computing WLSE algorithm.

In order to keep proper function of control algorithm, it is necessary to synchronize the control structure with power grid voltage curve. When supposing purely sinusoidal voltage curve, we only need to know the moments when the voltage curve crosses the zero axes (moments of polarity change) and the voltage polarity in every half-period. The easiest way to follow this condition is to convert sinusoidal curve (power grid voltage) into the square shape with logical levels of 0 and 3V, otherwise log. 0 and log. 1 as well. Log. 0 corresponds to negative half-wave, log. 1 to positive. Signal edges indicate the zero crossings. The DSP processor these zero crossing by means of external interrupt. The polarity is evaluated from the zero crossing direction – rising or falling edge [6].

Experiments were performed with the parameters of controllers given in Table 1. The switching frequency was set to be 5kHz and the parameters of the phase angle estimator were selected such that $\lambda = 0.999$ and $\gamma = 2$.

<table>
<thead>
<tr>
<th>Parameters of the controllers</th>
<th>frequency</th>
<th>voltage</th>
<th>current</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_P$</td>
<td>1</td>
<td>$K_P$</td>
<td>1.05</td>
</tr>
<tr>
<td>$K_I$</td>
<td>5</td>
<td>$K_I$</td>
<td>0.005</td>
</tr>
<tr>
<td>$K_{PI}$</td>
<td>$K_{PI}$</td>
<td>3000</td>
<td></td>
</tr>
<tr>
<td>$K_{PI}$</td>
<td>60</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The conditions for test were: $U_{\text{in}} = 235V$, $U_{\text{DC}} = 300V$, the input inductor $L = 35mH$, the DC link capacitor $C = 3000\mu F$. The parameters of induction motor were: $P_n = 7.5kW$, $U_n = 380V$, $f = 50Hz$, $n = 1500rpm$ and number of poles $p = 4$.

The experimental tests of a single phase PWM rectifier has been performed by induction motor (IM) with motor and
generator (recuperation) operation modes. The recuperation has been realized by the commutation dynamometer with parameters \( P_n = 41kW, \ U_n = 380V, \ I_n = 145A, \ f = 50Hz, \ n = 2800\text{rpm} \). The results of experimental tests are shown in the following figures.

\[ U_{dc} = 2800 \]

\[ n = 2800 \text{rpm} \]

The results of transient responses (PWM rectifier) when DC-link voltage changes gradually and no-load is attached. The input current surge is reduced by the ramp of reference value of DC-link voltage. The required value \( U_{dc} \) is reached in time 0.6s.

The results of transient responses PWM rectifier and PWM inverter in motor mode when IM has load on \( M_1 = 16Nm \) is shown in Fig. 7. In this case the voltage is in the phase with the current (the value of power factor is 0.994) and ripple of DC-link voltage is \( \pm 0.3V \).

Fig. 8 shows the experimental results of transient responses (PWM rectifier and PWM inverter) in recuperation (generator) mode when IM has load on \( M_1 = 16Nm \). The voltage is in phase opposition with the current (the value of power factor is -0.995) and ripple of DC-link voltage is \( \pm 0.6V \).

Fig. 9 shows the last results of transient responses when operation mode is changed (motor \( \rightarrow \) generator) in time 0.28s. Rise of DC-link voltage is about 2.3\% (7V) from reference value (300V).

![Figure 6: Transient responses when DC-link voltage changes gradually](image)

![Figure 7: Transient responses (motor mode): load on \( M_1 = 16Nm \) (input voltage, current, DC-link voltage and power factor)](image)

![Figure 8: Transient responses (generator mode): load on \( M_1 = 16Nm \) (input voltage, current, DC-link voltage and power factor)](image)

![Figure 9: Transient responses (motor mode \( \rightarrow \) generator mode): (input voltage, current, DC-link voltage and power factor)](image)

IV. CONCLUSION

This paper presented an advanced control structure design for a single phase PWM rectifier. The control structure consists of a proportional-resonant controller using a fast phase angle and frequency estimator. The estimation algorithm is derived from the weighted least-squares estimation method.

The PWM rectifier can perform well in many applications, for example as an active filter or as an input rectifier for an indirect frequency converter. This application is useful mainly in traction, where the AC voltage from the trolley wire is first rectified, and the traction inverters and also other auxiliary converters are fed from the output of the rectifier. A traction vehicle equipped with a PWM rectifier does not consume reactive power, will not load the supply network with harmonics and can recuperate. The proposed control structure is confirmed by experimental tests performed on designed laboratory prototype.
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