# ROBUST OUTPUT FEEDBACK AFFINE QUADRATIC CONTROLLER DESIGN

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Abstract: The paper deals with the robust output feedback controller design with guaranteed cost and affine quadratic stability for linear continuous time systems. The proposed approach leads to an iterative LMI based algorithm. Numerical examples are studied to verify effectiveness of the proposed method. The obtained results are compared with some other design procedures for robust affine quadratic controller design and robust parameter-dependent quadratic stability. 

Keywords: Robust control, Affine quadratic stability, Linear matrix inequality, Output feedback.

## 1. INTRODUCTION

During the last two decades numerous papers dealing with the design of robust output feedback control schemes to stabilize such systems have been published (Goh *et al.*, 1995; Iwasaki *et al.,* 1994; Cao *et al.,* 1998; Veselý, 2002). Various approaches have been used to study the two aspects of the robust stabilization problem, namely conditions under which the linear system described in state space can be stabilized via output feedback and the respective procedure to obtain a stabilizing or robustly stabilizing control law.

The necessary and sufficient conditions to stabilize linear continuous time invariant systems via static output feedback can be found in (Kučera and de Souza, 1995) and in (Veselý, 2001). In the above and other papers, authors basically conclude that despite the availability of many approaches and numerical algorithms the static output feedback problem is still open.

Recently, it has been shown that an extremely wide array of robust controller design problems can be reduced to the problem of finding a feasible point under a Biaffine Matrix Inequality (BMI) constraint. The BMI has been introduced in (Goh *et al.,* 1995). In this paper, the BMI problem of robust controller design with output feedback is reduced to an LMI problem (Boyd *et al.,* 1994). The theory of linear matrix inequalities (LMI) has been used to design robust output feedback controllers in several design methods (Henrion *et al.,* 2002; Veselý, 2001). Most of above works present iterative algorithms in which solution of a set of equations or set of LMI problems is repeated until certain convergence criteria are met.

Description of uncertain linear systems with affine parameter uncertainties and comprehensive development using the notion of quadratic stability by LMI approach are presented in (Boyd *et al.,* 1994). To reduce quadratic stability conservatism in robust controller design procedure the affine parameter dependent Lyapunov function has been introduced (Gahinet *et al.,* 1996; Yang and Lum, 2005) which is less conservative than quadratic stability. Recently, more general parameter-dependent Lyapunov functions have been exploited to develop less conservative robust stability criteria (de Oliveira *et al.,* 2000; Peaucelle *et al.*, 2000; Henrion *et al.*, 2002; Grman *et al.*, 2004).

This paper addresses the robust controller design problem with a new guaranteed cost function for a class of uncertain linear time-invariant systems, where the system matrices affinely depend on the uncertain parameters. Affine parameter-dependent Lyapunov functions are exploited to robust controller design, which guarantee affine quadratic stability in terms of linear matrix inequalities.

The proposed robust controller design method is based on the robust stability analysis results of (Peaucelle *et al.,* 2000), modified for robust controller design with guaranteed cost. Obviously, a performance of LQR problem includes state and control vectors. In the paper a new LQR criterion is proposed, which includes the vectors: of state derivative, state and control. This criterion allows taking into account some constraints on the rate of state variable changes.

The aims of this paper are the following:
1) to investigate the new guaranteed cost function in the robust affine controller design and
2) to provide a numerical comparison of the proposed robust affine controller design procedure with the following methods:
   - the affine controller design method based on the affine robust stability analysis proposed by (Gahinet *et al.,* 1996) using linearization approach (Han and Skelton, 2003);
   - the affine controller design method based on robust stability analysis proposed by (Peaucelle *et al.,* 2000) using linearization approach (Han and Skelton, 2003) and the new convexification function;
   - the parameter dependent Lyapunov function robust controller design method based on the robust stability analysis proposed by (Peaucelle *et al.,* 2000) using linearization approach (Han and Skelton, 2003).

The paper is organized as follows. In Section 2 the problem formulation and some preliminary results are presented. Main results are given in Section 3. In Section 4 the obtained theoretical results are applied to some examples.

## 2. PROBLEM FORMULATION AND PRELIMINARIES

In the context of robustness analysis and synthesis of robust controller for linear time invariant systems the following uncertain model is commonly used

$$\begin{aligned} \dot{x}(t) &= A(\theta)x(t) + B(\theta)u(t) \\ y(t) &= C(\theta)x(t), \qquad x(0) = x_0 \end{aligned} \tag{1}$$

where $x(t) \in R^{n}$, $u(t) \in R^{m}$ and $y(t) \in R^{l}$ are the state, control and output vectors, respectively; $\theta = [\theta_1, \ldots, \theta_p] \in R^{p}$ is the vector of uncertain and possibly time varying real parameters; $A(\theta)$, $B(\theta)$ and $C(\theta)$ are matrices of appropriate dimensions affinely depending on $\theta$

$$\begin{aligned} A(\theta) &= A_0 + A_1\theta_1 + A_2\theta_2 + \ldots + A_p\theta_p \\ B(\theta) &= B_0 + B_1\theta_1 + B_2\theta_2 + \ldots + B_p\theta_p \\ C(\theta) &= C_0 + C_1\theta_1 + C_2\theta_2 + \ldots + C_p\theta_p \end{aligned} \tag{2}$$

where $A_0, \ldots, A_p, B_0, \ldots, B_p, C_0, \ldots, C_p$ are known fixed matrices. Note, that in order to keep the polytope affine property, the matrix $B(\theta)$ or $C(\theta)$ has to be precisely known. In the following we assume that $C(\theta)$ is known and equal to the matrix *C*. In general, a polytope description of uncertainties results in a less conservative controller design than other characterizations of uncertainty (Boyd *et al.*, 1994).

The closed loop system (1) with the control algorithm

$$u(t) = FCx(t) \tag{3}$$

is given as follows

$$\begin{aligned} \dot{x}(t) &= (A(\theta) + B(\theta)FC)x(t) = \\ &= (A_{c0} + A_{c1}\theta_1 + \ldots + A_{cp}\theta_p)x(t) \\ &= A_c(\theta)x(t) \qquad x(0) = x_0 \end{aligned} \tag{4}$$

The system represented by (4) is a polytope of linear affine systems, which can be described by a list of its vertices

$$\dot{x}(t) = D_{ci}x(t), \qquad i = 1,2,\ldots,N \tag{5}$$

where $N = 2^{p}$. The system represented by (5) is quadratically stable if and only if there exists a Lyapunov matrix $P > 0$ such that

$$D_{ci}{}^{T}P + PD_{ci} < 0, \quad i = 1,2,\ldots,N \tag{6}$$

A weakness of quadratic stability is that it guards against arbitrary fast parameter variations. As a result, this test tends to be conservative for constant or slow-varying parameters $\theta$. To reduce conservatism when (4) is affine in $\theta$ and the parameters of system are time invariant, in (Gahinet *et al.,* 1996), the parameter-dependent Lyapunov function $P(\theta)$ has been used in the form

$$P(\theta) = P_0 + P_1\theta_1 + \ldots + P_p\theta_p \tag{7}$$

A robust controller design procedure with guaranteed cost and affine quadratic stability based on (Gahinet *et al.,* 1996) has been proposed in (Veselý, 2002). In this paper, we pursue the idea of (Peaucelle *et al.,*

2000) and introduce a new robust affine controller LMI design procedure with guaranteed cost.

The following definition and theorem by (Gahinet *et al.,* 1996) will be heavily exploited in the next development.

*Definition 1.* The linear system (4) is affine quadratically stable if there exist $p$+1 symmetric matrices $P_0$, $P_1$, ..., $P_p$ such that

$$P(\theta) = P_0 + P_1\theta_1 + \ldots + P_p\theta_p > 0 \qquad (8)$$

and

$$V(x,\theta) = x(t)^T P(\theta)x(t) > 0 \qquad (9)$$

$$\frac{dV(x,\theta)}{dt} = \qquad (10)$$
$$= x(t)^T \left( A_c(\theta)^T P(\theta) + P(\theta)A_c(\theta) + \frac{dP(\theta)}{dt} \right) x(t) < 0$$

for $\theta = \lfloor \theta_1, \ldots, \theta_p \rfloor$. □

Note that the quadratic stability corresponds to the case when $P_1 = P_2 = \ldots = P_p = 0$. The sufficient affine quadratic stability conditions are given in the next theorem (Gahinet *et al.,* 1996).

*Theorem 1.* Consider a linear system governed by (4), where $A_c(\theta)$ depends affinely on the uncertain parameter vector $\theta = \lfloor \theta_1, \ldots, \theta_p \rfloor$ and $\theta_i$ satisfies

$$\theta_i \in \langle \underline{\theta}_i, \overline{\theta}_i \rangle, \quad \dot{\theta}_i \in \langle \underline{\nu}_i, \overline{\nu}_i \rangle \qquad \text{for } i = 1,2,\ldots,p \quad (11)$$

where $\underline{\theta}_i, \overline{\theta}_i, \underline{\nu}_i, \overline{\nu}_i$ are known lower and upper bounds. Let $\Gamma$ and $\Lambda$ denote sets of $N = 2^p$ vertices of the parameters box (11) and the rate of variation box (11), respectively

$$\begin{aligned} \Gamma &= \{(\gamma_1, \gamma_2, \ldots, \gamma_p) : \gamma_i = \underline{\theta}_i \quad or \quad \gamma_i = \overline{\theta}_i\} \\ \Lambda &= \{(\lambda_1, \lambda_2, \ldots, \lambda_p) : \lambda_i = \underline{\nu}_i \quad or \quad \lambda_i = \overline{\nu}_i\} \end{aligned} \qquad (12)$$

and let

$$\theta_m = \left[ \frac{\underline{\theta}_1 + \overline{\theta}_1}{2}, \ldots, \frac{\underline{\theta}_p + \overline{\theta}_p}{2} \right] \qquad (13)$$

denote the average value of the parameter vector.

This system is affine quadratically stable if $A_c(\theta_m)$ is stable and if there exist $p$+1 symmetric matrices $P_0, \ldots, P_p$ such that $P(\theta) > 0$ satisfies

$$L(\gamma,\lambda) = A_c(\gamma)^T P(\gamma) + P(\gamma)A_c(\gamma) + P(\lambda) - P_0 < 0 \qquad (14)$$

for all $(\gamma,\lambda) \in \Gamma \times \Lambda$ and

$$A_{c_i}^T P_i + P_i A_{c_i} \geq 0 \qquad \text{for } i = 1,2,\ldots,p \qquad (15)$$

where $A_{c_i} = A_i + B_i FC$. □

For scalar quadratic function the implications of multiconvexity are clarified by the next lemma (Gahinet *et al.,* 1996).

*Lemma 1.* Consider a scalar quadratic function of $\theta \in R^{\,p}$

$$f(\theta_1, \ldots, \theta_p) = \alpha_0 + \sum_i \alpha_i \theta_i + \sum_{i<j} \beta_{ij}\theta_i\theta_j + \sum_i \omega_i \theta_i^2 \qquad (16)$$

and assume that $f(.)$ is multiconvex, that is

$$\frac{\partial^2 f(\theta)}{\partial \theta_i^2} = 2\omega_i \geq 0 \qquad \text{for } i = 1,\ldots,p \qquad (17)$$

Then $f(.)$ is negative in the uncertain box (12) if and only if it takes negative values at the corners of (11); that is, if and only if $f(\gamma) < 0$ for all $\gamma$ in the vertex set $\Gamma$ given by (12). □

The following new performance index is associated with the system (1)

$$J = \int_0^\infty \begin{bmatrix} x^T & \dot{x}^T & u^T \end{bmatrix} \begin{bmatrix} Q & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ u \end{bmatrix} dt \qquad (18)$$

where $Q = Q^T \geq 0$, $S = S^T \geq 0$, $R = R^T > 0$ are matrices of compatible dimensions.

The problem studied in this paper can be formulated as follows: For a continuous time system described by (1) design a static output feedback controller with the gain matrix $F$ and the control algorithm (3) such that the closed loop system (4) is affine quadratic stable with guaranteed cost.

*Definition 2.* Consider the system (1). If there exist a control law $u^*$ and a positive scalar $J^*$ such that the closed loop system (4) is stable and the closed loop value cost function (18) satisfies $J \leq J^*$, then $J^*$ is said to be the guaranteed cost and $u^*$ is said to be the guaranteed cost control law for the system (1). □

## 3. MAIN RESULTS

In this section we present a new procedure to design a static output feedback controller for affine continuous time linear systems (1), which ensures the guaranteed cost and affine quadratic stability of the closed loop system. The following theorem is one of the main results.

*Theorem 2.* Consider the system (1) and the Lyapunov function $V(\theta)=x(t)^T P(\theta)x(t)>0$, $P(\theta)$ described by (8). The following statements are equivalent:

- There exist matrices $Q=Q^T \geq 0$, $S=S^T \geq 0$, $R=R^T >0$ and a matrix *F*, such that system (1) is static output feedback affine quadratic stabilizable (AQS) with the guaranteed cost

$$\int_0^\infty x(t)^T \left(Q+C^T F^T RFC+A_c(\theta)^T SA_c(\theta)\right)x(t)dt \leq \\ \leq x_0{}^T P(\theta)x_0 \tag{19}$$

where $A_c(\theta)=A(\theta)+B(\theta)FC$.

- There exist $p$+1 symmetric matrices $P_0,P_1,\ldots,P_p$ that $P(\theta)>0$, symmetric matrices $Q=Q^T \geq 0$, $S=S^T \geq 0$, $R=R^T >0$ and matrices *F*, $N_1$ and $N_2$ such that the following inequality holds

$$\begin{bmatrix} \Psi_1 & P(\theta)+N_1{}^T+A_c(\theta)N_2 \\ P(\theta)+N_1+N_2{}^T A_c(\theta) & S-N_2^T-N_2 \end{bmatrix}<0 \tag{20}$$

where

$$\Psi_1=-N_1{}^T A_c(\theta)-A_c(\theta)^T N_1+Q+C^T F^T RFC+\dot{P}(\theta)$$

- There exist $p$+1 symmetric matrices $P_0,P_1,\ldots,P_p$ that $P(\theta)>0$, symmetric matrices $Q=Q^T \geq 0$, $S=S^T \geq 0$, $R=R^T >0$ and matrices *F*, $N_1$ of compatible dimensions such that the following two inequalities hold

$$\begin{bmatrix} \Psi_2 & A_{ci}{}^T \\ A_{ci} & -S^{-1} \end{bmatrix}<0 \tag{21}$$

$$-N_1{}^T A_{ci}-A_{ci}^T N_1+Q+C^T F^T RFC+\dot{P}(\theta^i)<0 \tag{22}$$

where $\Psi_2=A_{ci}^T P(\theta^i)+P(\theta^i)A_{ci}+Q+C^T F^T RFC+\dot{P}(\theta^i)$

$A_{ci}=A_{vi}+B_{vi}FC$ $i=1,2,\ldots,N$, $N=2^p$

$A_{vi},B_{vi}$ - are vertices of system (1);

$P(\theta^i)=P_0+\sum_{j=1}^{p} P_j\theta_j$ and $\theta_j$ has the value corresponding to i-th vertex. □

Proof. (Sketch)
Due to Lemma 1 and using the Elimination Lemma (Skelton *et al.,* 1997) to eliminate the matrix $N_2$ from (20), the inequalities (21) and (22) are obtained which proves that the third and second statements are equivalent. The sufficiency of second to first statement, will be prove in the next. Let (20) holds. If one left multiplies (20) by the matrix $\begin{bmatrix} I & A_c^T(\theta) \end{bmatrix}$ and right multiplies by its transpose (Geromel *et al.,* 1998) the following inequality is obtained

$$A_c^T(\theta)P(\theta)+P(\theta)A_c(\theta)+Q+C^T F^T RFC+ \\ +\dot{P}(\theta)+A_c^T(\theta)SA_c(\theta)<0 \tag{23}$$

Due to Definition 1 and (23)

$$\frac{dV(x,\theta)}{dt}< \\ <-x(t)^T \left(Q+C^T F^T RFC+A_c^T(\theta)SA_c(\theta)\right)x(t)<0 \tag{24}$$

Therefore the closed loop system is asymptotically stable. Furthermore, by integrating both sides of (24) from 0 to *T* and using the initial conditions $x_0$, we obtain

$$V(0)-V(T)\geq \\ \int_0^T x(t)^T \left(Q+C^T F^T RFC+A_c^T(\theta)SA_c(\theta)\right)x(t)dt \tag{25}$$

As the closed loop system is asymptotically stable, with $T\to\infty$

$$V(T)=x(T)^T P(\theta)x(T)\to 0 \tag{26}$$

Hence, we get the inequality (19), which proves the sufficiency statement for the one. □

In the Theorem 2 the matrix inequalities (21) and (22) are nonconvex. There are following possible approaches to solve this nonconvex problem in a computationally efficient way using the linearization (Han and Skelton, 2003) and the convexifying algorithm (de Oliveira *et al.,* 2000). We use both algorithms. A convexifying algorithm requires a convexifying potential function. There can exist many candidates for the convexifying potential function for a given nonconvex matrix inequality. The following theorem is the next main result presenting the new convexification functions.

*Theorem 3.* Let a symmetric, positive definite matrix $X\in R^{n\times n}$ is given. Then the conditions a) and b) in each of the following cases 1. - 4. are equivalent:

1. a)

$$\begin{bmatrix} L_{1i}-ZB_{vi}R^{-1}B_{vi}^T X-XB_{vi}R^{-1}B_{vi}^T Z & XB_{vi}R^{-1} \\ R^{-1}B_{vi}^T X & -R^{-1} \end{bmatrix}<0$$

$$\text{b)}\quad \begin{bmatrix} L_{1i} - ZB_{vi}R^{-1}B_{vi}^T Z & (X-Z)B_{vi}R^{-1} \\ R^{-1}B_{vi}^T(X-Z) & -R^{-1} \end{bmatrix} < 0 \tag{27}$$

2. a)

$$\begin{bmatrix} L_{1i} - ZB_{vi}R^{-1}B_{vi}^T - B_{vi}R^{-1}B_{vi}^T Z & B_{vi}R^{-1} \\ R^{-1}B_{vi}^T & -R^{-1} \end{bmatrix} < 0$$

$$\text{b)}\quad \begin{bmatrix} L_{1i} - ZB_{vi}R^{-1}B_{vi}^T Z & (I-Z)B_{vi}R^{-1} \\ R^{-1}B_{vi}^T(I-Z) & -R^{-1} \end{bmatrix} < 0 \tag{28}$$

3. a)

$$\begin{bmatrix} L_{1i} & 0 \\ 0 & -R^{-1} \end{bmatrix} < 0$$

$$\text{b)}\quad \begin{bmatrix} L_{1i} - ZB_{vi}R^{-1}B_{vi}^T Z & -ZB_{vi}R^{-1} \\ -R^{-1}B_{vi}^T Z & -R^{-1} \end{bmatrix} < 0 \tag{29}$$

where $L_{1i} = A_{vi}^T P(\theta^i) + P(\theta^i)A_{vi} + Q$ and $Z = P(\theta^i)$ ($X = X(\theta^i)$), $Z = -N_1^T$ respectively, $i = 1,2,\ldots,N$. □

Proof. Because

$$\det\begin{bmatrix} I_1 & P_i B_{vi} \\ 0 & I_2 \end{bmatrix} \neq 0 \tag{30}$$

the following equation is a congruence transformation (Ayres, 1962) for the matrix on the left hand side of inequality (28a). If we left multiply the matrix (28a) with matrix in (30) and right multiply by its transpose we obtain

$$\begin{bmatrix} I_1 & ZB_{vi} \\ 0 & I_2 \end{bmatrix}\begin{bmatrix} L_{1i} - ZB_{vi}R^{-1}B_{vi}^T - B_{vi}R^{-1}B_{vi}^T Z & B_{vi}R^{-1} \\ R^{-1}B_{vi}^T & -R^{-1} \end{bmatrix}\cdot \begin{bmatrix} I_1 & 0 \\ B_{vi}^T Z & I_2 \end{bmatrix} = \begin{bmatrix} L_{1i} - ZB_{vi}R^{-1}B_{vi}^T Z & (I-Z)B_{vi}R^{-1} \\ R^{-1}B_{vi}^T(I-Z) & -R^{-1} \end{bmatrix} < 0 \tag{31}$$

which is the inequality (28b). Similarly other cases can be proven. □

## 4. EXAMPLES

*Example 1* has been borrowed from (Geromel *et al.*, 1996) to demonstrate the use of algorithm given by (21) and (22) for the affine controller design using the linearization approach (PEAU-AQ_lin) and the new convexification function (PEAU-AQ_con). To investigate the influence of the new guaranteed cost function (18) on closed-loop eigenvalues for the above control design methods are depicted in Fig. 1. Increasing the entries of the weighting matrix $S$ degrades the robustness margin but improves the performance.

*Example 2.* Assume the affine model (1) modified so that all uncertainties in (1) are non-dimensional and normalized so that $-1 \le \theta_1 \le 1$, $i = 1,\ldots,p$ while matrices $A_{ck}, k = 0,\ldots,p$ in (4) are constant. The

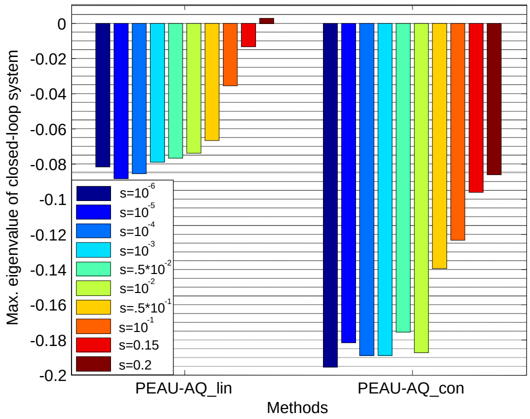


Fig. 1. Results in terms of maximal closed-loop eigenvalue for affine robust controller design in dependence on entries of weighting matrix $S$ in guaranteed cost function.

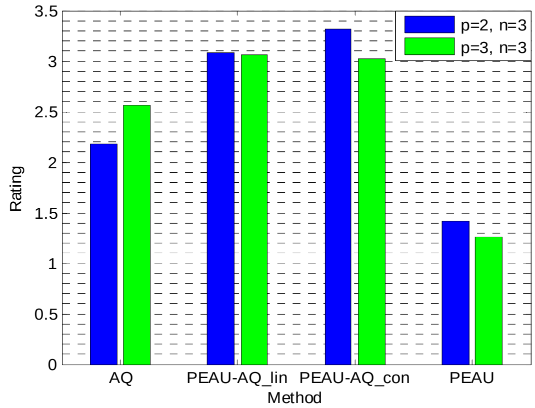


Fig. 2. Results of robust stability evaluation in terms of rating ("the lower value –the better method").

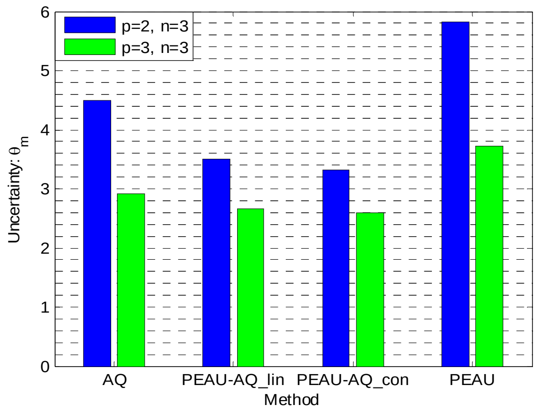


Fig. 3. Results of robust controller design method evaluating in terms of $\theta_m$ ("the higher value-the better method").

matrices $A_{vi}, B_{vi}, i = 0,1,\ldots,N$ are obtained considering $\theta_i = \pm 1, i = 1,\ldots,p$. The following test has been applied to 50 stable affine systems (1) generated for 1) $p = 2$ and $A_{ck}$ of the size $n = 3$; 2) $p = 3$ and $n = 3$. In each example, the maximum value of the uncertainty parameter $\theta_m$ has been evaluated for the following affine controller design methods:

- AQ - based on (Gahinet *et al.,* 1996) using linearization approach (Han and Skelton, 2003);
- PEAU-AQ_lin - based on (Peaucelle *et al.,* 2000) using linearization approach (Han and Skelton, 2003);
- PEAU-AQ_con - based on (Peaucelle *et al.,* 2000) using new convexification function (27a);
- PEAU - the parameter dependent Lyapunov function robust controller design method based on (Peaucelle *et al.,* 2000) using linearization approach (Han and Skelton, 2003).

The obtained results have been evaluated as follows:

1. All methods have been applied to each example, ranged with respect to the maximum value of uncertainty parameter $\theta_{mi}, i = 1,...,50$ and evaluated in terms of points assigned according to the rating, (i.e. the highest value of $\theta_{mi}$ - best rating = 1 point, … etc), i.e. the fewer points, the better rating of the respective method (in Fig. 2).
2. For each method, the mean value $\theta_{me}$ of all maximum uncertainty parameters $\theta_{mi}$ achieved has been computed $\theta_{me} = \left(\sum_{i=1}^{50} \theta_{mi}\right)/50$ and the methods were ranged according to decreasing values of $\theta_{me}$. Hence, in this case, the higher value of $\theta_{me}$, the better rating of the respective method (in Fig. 3).

## 5. CONCLUSION

In this paper, we have proposed a new method for designing affine robust controllers with output feedback via LMI approach, which guarantees the value of new cost function. The design procedure is based on the robust stability analysis results, modified for the affine robust controller design and belonging to BMI problems. We have reduced the BMI problem to an LMI one using the linearization approach and the new convexification function. The results obtained by a thorough numerical verification on examples show the effectiveness of the proposed methods.

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