ROBUST OUTPUT FEEDBACK CONTROLLER DESIGN BY QUADRATIC STABILITY METHODS

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Abstract: The paper provides survey of some recent quadratic stability methods for static output feedback robust controller design for linear continuous-time invariant systems with convex polytopic uncertainty and their mutual comparison. Robust controller design is based on linear matrix inequalities (LMIs) conditions and single Lyapunov functions. The presented quadratic stability methods are compared on numerical examples and randomly generated ones and it is shown which of them provides the less conservative results.

Keywords: quadratic stability, linear matrix inequality, robust controller, output feedback

1 INTRODUCTION

One of the most challenging problems in control theory remains to find numerically tractable necessary and sufficient conditions for the stabilizability of linear time-invariant (LTI) systems via static output feedback. The output feedback problem is one of the most important open questions of control engineering. In a simple way, the problem can be formulated as follows: for a given complex linear system a robust controller with a static output feedback is to be found, which would provide some desirable characteristics to the closed-loop systems, or determine that such a feedback does not exist.

Lyapunov functions have been used in the study of stability of dynamic systems since many years ago. Concerning linear systems with uncertain parameters, the use of Lyapunov functions has allowed important developments which are mainly related to the concept of quadratic stability and convex optimization applied to robust control problems during the last two decades. Thanks to quadratic stability, the stability of a polytope of matrices can be attested by means of convex feasibility test performed only at the vertices of the uncertainty domain, which means a feasibility test of a set of linear matrix inequalities (LMIs). A drawback of quadratic stability is that it guards against arbitrary fast parameter variations and thus it uses a single Lyapunov function for testing over the whole uncertainty box [Boyd *et al.*, 1994].

The aim of this paper is to provide only a numerical comparison among three LMI based quadratic methods for the robust stability of uncertain linear systems in polytopic domains: the V-F iteration method [El Ghaoui and Balakrishnan, 1994], the two-step method [Veselý, 2001] and the linearization method [Cao *et al*., 1998]. The V-F iteration algorithm is based on an alternative solution of two convex LMI optimization problems obtained by fixing the Lyapunov matrix or the gain controller matrix. The two-step method does not require iteration of LMI problems and the linearization method uses iterative LMI algorithm. An aspect of conservatism is investigated on numerical examples and randomly generated ones. The proposed LMI based algorithms are computationally simple and tightly connected with the Lyapunov function, quadratic stability, guaranteed cost and LQ optimal state feedback design.

2 PROBLEM FORMULATION AND PRELIMINARIES

Consider the following linear continuous time-invariant uncertain system

$$\begin{aligned} \dot{x}(t) &= (A+\delta A)x(t)+(B+\delta B)u(t) \\ y(t) &= Cx(t), \qquad\qquad x(0)=x_0 \end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^m$ and $y \in R^l$ are state, control and output vectors, respectively; $A \in R^{n\times n}$, $B \in R^{n\times m}$ and $C \in R^{l\times n}$ are known matrices of appropriate dimensions; δA, δB are unknown but norm bounded uncertainties. In the next development the matrix affine type uncertain structure will be used

$$\begin{aligned} &\delta A = \sum_{j=1}^{p} \varepsilon_j A_j\,, \quad \delta B = \sum_{j=1}^{p} \varepsilon_j B_j \\ &\underline{\varepsilon}_j \le \varepsilon_j \le \overline{\varepsilon}_j\,, \quad j = 1,2,\ldots,p \end{aligned} \tag{2}$$

where A_j, B_j are known matrices; ε_j are uncertain parameters ($\underline{\varepsilon}_j$, $\overline{\varepsilon}_j$ are known lower and upper uncertainty bounds). In general, the polytope characterization of uncertainties results in less conservative controller designs than using other characterizations of uncertainty [Boyd *et al.,* 1994].

The problem studied in this paper can be formulated as follows. For linear continuous time-invariant system described by (1) a robust static output feedback controller is to be designed for the control algorithm

$$u = FCx\,, \quad F \in R^{m\times l} \tag{3}$$

such that the closed loop system

$$\dot{x} = (A + BFC)x + (\delta A + \delta BFC)x \tag{4}$$

is stable for all admissible uncertainties described by (2) and simultaneously guaranteeing the suboptimal solution to the performance index

$$J = \int_0^\infty \left(x^T Q x + u^T R u\right) dt \tag{5}$$

where $Q = Q^T \ge 0$ and $R = R^T > 0$ are matrices of compatible dimensions, $Q \in R^{n\times n}$, $R \in R^{m\times m}$.

The nominal model of the system (1) is

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{6}$$

The following lemma is well known [Lankaster, 1969].

Lemma 1

Suppose $P > 0$ to be the solution of the following Lyapunov matrix equation

$$A^T P + PA + Q = 0 \tag{7}$$

Then A is stable iff $P > 0$ and $Q > 0$.

If such P exists, we say that the matrix A is quadratically stable. A linear time invariant system is stable if and only if it is quadratically stable. It is possible, however, that e.g. linear polytopic systems can be stable without being quadratically stable [Boyd *et al.,* 1994].

The closed loop polytopic system with output feedback algorithm (3) can be described by the list of its vertices ($N = 2^p$)

$$\dot{x} = (A_{vi} + B_{vi}FC)x = A_{ci}x\,, \quad i = 1,2,\ldots,N \tag{8}$$

The linear uncertain system (4) belongs to a convex polytopic set defined as

$$\dot{x} = A(\alpha)x \tag{9}$$

whereby

$$S := \left\{ A(\alpha) : A(\alpha) = \sum_{i=1}^{N} \alpha_i A_{ci}\,,\ \sum_{i=1}^{N} \alpha_i = 1, \quad \alpha_i \geq 0 \right\}. \tag{10}$$

Lemma 2

The system represented by (9) is quadratically stable if and only if there is a common Lyapunov matrix $P > 0$ such that

$$A_{ci}^T P + PA_{ci} < 0\,, \quad i = 1,2,\ldots,N \tag{11}$$

3 OUTPUT FEEDBACK CONTROLLER DESIGN

In this section we present three quadratic stability methods (the V-F iteration method, the two-step method and the linearization method) to design a static output feedback controller for linear continuous time-invariant systems (1) which ensures the guaranteed cost (5) of the closed loop system.

3.1 V-F iteration method

Inequalities (11) can be extended and modified to the form

$$(A_{vi} + B_{vi}FC)^T P + P(A_{vi} + B_{vi}FC) + Q + C^T F^T RFC < 0\,, \quad i = 1,2,\ldots,N \tag{12}$$

In the system of inequalities (12) the positive definite matrix P and the feedback gain F are unknown.

If such matrices exist then the polytopic system is quadratically stable and simultaneously the matrix F ensures a minimum value of the quadratic performance criterion (5).

For the functional (5) the following inequality holds

$$\int_0^\infty \left(x^T Qx + u^T Ru\right)dt < x_0{}^T Px_0 \tag{13}$$

where the matrix P is the solution to inequalities (12) and x_0 is initial condition. With regard to matrices P and F a solution to (12) belongs to a class of bilinear matrix inequalities (BMI) and in the case of a convex problem (e.g. when matrix F is known) to the class of linear matrix inequalities (LMI). In general a modification of nonlinear (convex) inequalities to the LMI form employs the Schur complement [Boyd *et al.*, 1994], [Hypiusová, 2002] i.e. for any matrices E_{11}, E_{22} and E_{12} where E_{11} and E_{22} are symmetric the following statements are equivalent:

a) $\begin{bmatrix} E_{11} & E_{12} \\ E_{12}^T & E_{22} \end{bmatrix} > 0$ (14)

b) if $E_{22} > 0$: $E_{11} - E_{12}E_{22}^{-1}E_{12}^T > 0$ (15)

c) if $E_{11} > 0$: $E_{22} - E_{12}^T E_{11}^{-1} E_{12} > 0$. (16)

Next we present solution to (12) by the V-F iteration. Its principle consists in alternately solving two convex LMI optimization problems, where in the first problem we compute the matrix P for a fixed F and in the second problem vice versa.

The first task is to find the matrix F for which systems $A_{vi} + B_{vi}FC$, $i = 1,2,\ldots,N$ are all stable. This matrix determines initial condition for the problem solution.

Algorithm 1

1. $j = 1, F = F_0$ (For stable matrices A_{vi}, $F_0 = 0$)
2. Using the LMI algorithm compute the matrix P_j from the following inequalities

$$\begin{aligned} &(A_{vi} + B_{vi}F_{j-1}C)^T P_j + P_j(A_{vi} + B_{vi}F_{j-1}C) + Q + C^T F_{j-1}^T R F_{j-1} C < 0, \quad i = 1,\ldots,N \\ &0 < P_j < \rho I, \quad \rho > 0 \end{aligned} \tag{17}$$

   where ρ is a given positive upper bound for the maximal eigenvalue of P_j.
3. For the known matrix $P_j > 0$ compute F_j using LMI

$$\begin{bmatrix} (A_{vi} + B_{vi}F_jC)^T P_j + P_j(A_{vi} + B_{vi}F_jC) + Q & C^T F_j^T R \\ RF_jC & -R \end{bmatrix} < 0, \quad i = 1,\ldots,N \tag{18}$$

4. Compute $er = \|F_j - F_{j-1}\|$.
   If $er \le tolerance$ stop, else $j = j + 1$ and go to Step 2.

The V-F iteration algorithm is guaranteed to converge, but not necessarily to the global optimum of the problem depending on the starting conditions.

3.2 Two-step method

Consider the following algebraic Riccati inequalities

$$A_{vi}^T P + PA_{vi} - PB_{vi}R^{-1}B_{vi}^T P + Q < 0 \tag{19}$$

Define $P = S^{-1}$. Using the Schur complement formula (14) and (15) the inequality (19) is equivalent to the following linear matrix inequality

$$\begin{bmatrix} SA_{vi}^T + A_{vi}S - B_{vi}R^{-1}B_{vi}^T & S \\ S & -Q \end{bmatrix} < 0\,, \qquad \gamma I < S\,, \qquad i = 1,2,\ldots,N \tag{20}$$

where $\gamma \geq 0$ is some non-negative constant.

Inequalities (12) can be extended and modified to the form

$$A_{vi}^T P + PA_{vi} + Q + C^T F^T B_{vi}^T P + PB_{vi}FC + C^T F^T RFC < 0\,, \qquad i = 1,2,\ldots,N \tag{21}$$

and after some manipulation

$$-R + \left(RFC + B_{vi}^T P\right)G_i^{-1}\left(RFC + B_{vi}^T P\right)^T < 0\,, \qquad i = 1,2,\ldots,N \tag{22}$$

where $G_i = -\left(A_{vi}^T P + PA_{vi} - PB_{vi}R^{-1}B_{vi}^T P + Q\right)$.

With $P = S^{-1}$, inequality (22) can be rewritten using Schur complement as follows

$$\begin{bmatrix} -R & RFC + B_{vi}^T P \\ \left(RFC + B_{vi}^T P\right)^T & -G_i \end{bmatrix} < 0\,, \qquad i = 1,2,\ldots,N \tag{23}$$

The algorithm for static output feedback simultaneous stabilization for the system (9) with a guaranteed cost (13) using the non-iterative LMI approach is given as follows.

Algorithm 2

1. Using the LMI based algorithm calculate *S* from the inequality (20). $P = S^{-1}$.
2. Via the LMI based algorithm compute *F* from the inequality (23).
3. If the solution (20) is not feasible, the polytope system (8) is not simultaneously stabilizable and if (23) is not feasible (the closed loop system (8) is not stable) change *Q* and *R* or decrease $|\varepsilon_i|$, $i = 1,2,\ldots,p$.

If the solutions (20) and (23) are feasible with respect to *S* and *F* then the uncertain system (1) is quadratically stable with a guaranteed cost control algorithm $u = Fy$ and $J^* = x_0^T P x_0$ is the guaranteed cost for the uncertain closed loop system.

3.3 Linearization method

Inequalities (21) can be modified to the following quadratic matrix inequalities (QMIs)

$$A_{vi}^T P + PA_{vi} + Q - PB_{vi}R^{-1}B_{vi}^T P + \left(FC + R^{-1}B_{vi}^T P\right)^T R\left(FC + R^{-1}B_{vi}^T P\right) < 0\,, \quad i = 1,2,\ldots,N \tag{24}$$

If it is possible to find *P* > 0 and *F* satisfying the QMI in equation (24), then a stabilizing static output feedback gain exists. An advantage of this approach to obtain a stabilizing feedback gain *F* is that *F* is no longer assumed to be a function of the solution *P* of a special equation or inequality.

Due to the negative sign in the $-PB_{vi}R^{-1}B_{vi}^T P$ term, equations (24) cannot be simplified to LMI. To accommodate the $-PB_{vi}R^{-1}B_{vi}^T P$ term, we introduce an additional design variable *X*. By linearization using inequality $\left(X - P\right)^T B_{vi}R^{-1}B_{vi}^T\left(X - P\right) \geq 0$ for any *X* and *P* of the same dimension, we obtain

$$X^T B_{vi} R^{-1} B_{vi}^T P + P^T B_{vi} R^{-1} B_{vi}^T X - X^T B_{vi} R^{-1} B_{vi}^T X \le P^T B_{vi} R^{-1} B_{vi}^T P, \quad i = 1,2,\ldots,N \tag{25}$$

with equality holding for $X = P$. By combining inequalities (25) and (24), we obtain a sufficient condition for the existence of static output feedback matrix F given by

$$\begin{aligned} &A_{vi}^T P + PA_{vi} + Q - XB_{vi} R^{-1} B_{vi}^T P - PB_{vi} R^{-1} B_{vi}^T X + XB_{vi} R^{-1} B_{vi}^T X + \\ &+ \left(FC + R^{-1} B_{vi}^T P\right)^T R\left(FC + R^{-1} B_{vi}^T P\right) < 0 \end{aligned}, \quad i = 1,2,\ldots,N \tag{26}$$

Using the Schur complement (14) and (15), inequalities (26) are equivalent to the following QMIs

$$\begin{bmatrix} A_{vi}^T P + PA_{vi} + Q - XB_{vi} R^{-1} B_{vi}^T P - PB_{vi} R^{-1} B_{vi}^T X + XB_{vi} R^{-1} B_{vi}^T X & \left(FC + R^{-1} B_{vi}^T P\right)^T \\ FC + R^{-1} B_{vi}^T P & -R^{-1} \end{bmatrix} < 0 \tag{27}$$

These QMIs can be solved by an iterative approach. Namely, if X is fixed, then (27) reduces to LMI problem with unknown F and P. The LMI problem is convex and can be solved efficiently if a feasible solution exists.

Algorithm 3

1. Select $Q > 0$, and solve P from the following algebraic Riccati equation

   $$A^T P + PA - PBR^{-1} B^T P + Q = 0 \tag{28}$$

   Set $i = 1$ and $X = P$.
2. For the known matrix X compute F using LMI (27)
3. Compute $er = \|X - P\|$.

   If $er \le tolerance$ stop, else $i = i + 1$, $X = P$ and go to Step 2.

If the algorithm fails to arrive at a stabilizing solution, we may select another Q and run the LMI algorithm again. Our numerical experience indicates that the initial choice with $Q = I$ always leads to a convergent result.

4 EXAMPLES

4.1 Method of evaluation

In this section the properties and power of individual methods presented in Sections 3 have been tested on several examples taken from references and laboratory plants at our department as well as on 50 randomly generated ones. To be able to evaluate the conservatism of each particular method, the term "stability region size" has been adopted. In each tested example, it has been measured in terms of the parameter ε corresponding to the maximum uncertainty polytope for which the closed loop affine uncertain system with the gain matrix F still remains stable.

Consider the following closed loop polytopic system with output feedback algorithm (3) described by the list of its vertices ($N = 2^p$)

$$\dot{x} = \left(A_{vi} + B_{vi} FC\right)x = A_{ci} x, \quad i = 1,2,\ldots,N \tag{29}$$

The closed loop robust stability problem can be extended, as a question may arise about "how robust" the closed loop under the considered controller is:

What is the maximum range for uncertainty parameters such that the closed loop affine uncertain system with the gain matrix (29) remains stable?

$$\varepsilon = \max\left|\varepsilon_j\right|, \quad j = 1,2,\ldots,p \tag{30}$$

The following test has been applied:

- first, all considered methods were tested on ten continuous-time models taken from references and five laboratory plants at our department,
- then, 50 matrix A_{vi}, B_{vi} for closed loop polytopic system (29) were generated considering $\varepsilon_j \in \langle -1; 1 \rangle$, $j = 1,2,\ldots,p$ for the pairs ($n = 4$, $p = 2$) and ($n = 5$, $p = 2$); (the matrix C was constant: $C = [1\ 0\ 1\ 0;\ 0\ 1\ 0\ 1]$),
- finally, in each example, the maximum value of the uncertainty parameter ε was evaluated for each considered robust stability condition.

The obtained results have been evaluated as follows:

- For each example, all methods were arranged according to the maximum value of the uncertainty parameter ε and number of points assigned with respect to their priority (the highest value of ε - best rating – 1 point, ... etc.), i.e. the fewer points, the better rating of the respective method.
- For each method, the mean value ε_m of all uncertainty parameters obtained in the considered examples was computed, and the methods were arranged according to decreasing values of ε_m. Hence, in this case, the higher value of ε_m, the better rating of the respective method.

These two proposed criteria have been chosen due to their obvious interpretation. While the first criterion evaluates the method's priority ("the fewer points – the better method"), the second one estimates the "size" of the stability region ("the higher ε_m – the better method").

4.2 Results for real plants

All considered methods were tested on ten continuous-time models taken from references:

- [Benton and Smith, 1999], (p = 1, n = 4, m = 1, l = 2), (p = 2, n = 4, m = 2, l = 1),
- [Azuma *et al*., 2000], (p = 2, n = 3, m = 2, l = 2),
- [Takahashi *et al*., 2002], (p = 1, n = 3, m = 1, l = 3),
- [Cao and Sun, 1998], (p = 2, n = 3, m = 1, l = 2), (p = 1, n = 3, m = 1, l = 2),
- [Chilali *et al*., 1999], (p = 1, n = 5, m = 2, l = 3),
- [Veselý, 2000], (p = 1, n = 3, m = 1, l = 2),
- [Veselý *et al*., 2001], (p = 2, n = 3, m = 1, l = 2),
- [Veselý, 2001], (p = 2, n = 10, m = 2, l = 4),

and five laboratory plants at our department.

Results obtained for the above-considered pairs are summarized in Table 1 and the corresponding charts are in Fig. 1 and Fig. 2.

Table 1

Methods	V-F	TS	LIN
Rating	1.8667	2.4000	2.0000
q_m	1.3107	0.2468	1.3488

where the above acronyms have the following meaning:
V-F – V-F iteration method [Ghaoui and Balakrishnan, 1994], *Algorithm 1*
TS – two-step method [Veselý, 2001], *Algorithm 2*
LIN – linearization method [Cao *et al*., 1998], *Algorithm 3*

where rating and maximum uncertainty parameter q_m have been calculated using the mean values obtained from the 15 examples.

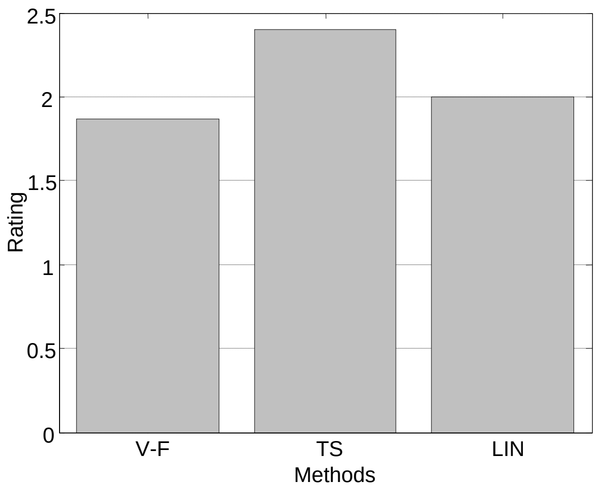

Fig. 1 – Result of robust stability evaluation in terms of rating

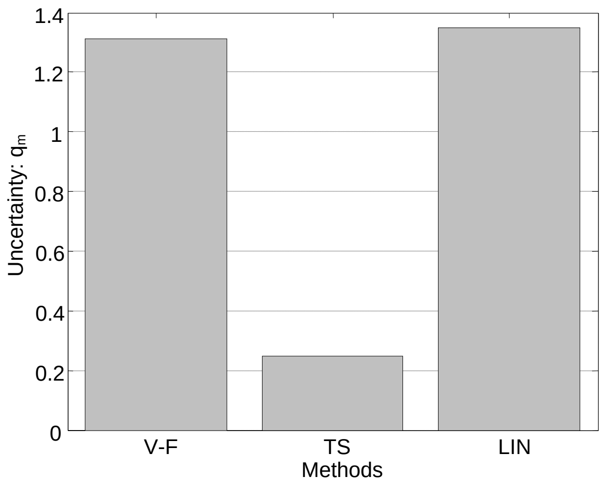

Fig. 2 – Result of robust stability evaluation in terms of uncertainty q_m

For all three quadratic methods (V-F, TS and LIN) the V-F iteration method is the less conservative: the method has obtained a minimum of points. The second place goes to the method LIN but the mean value q_m is larger than in other methods. On the average, for fifteen examples the two-step method is the most conservative with smallest mean value of uncertainty parameter q_m.

4.3 Results for generated examples

For 50 matrix A_{vi}, B_{vi} for closed loop polytopic system (29) were generated considering $\varepsilon_j \in \langle -1; 1 \rangle$, $j = 1,2,\dots,p$ for the pairs $(n = 4,\ p = 2)$ and $(n = 5,\ p = 2)$. The results have been evaluated as explained in Section 4.1.

Results obtained for the above-considered pairs are summarized in Table 2 and the corresponding charts are in Fig. 3 and Fig. 4.

Table 2

p	n		V-F	TS	LIN
2	4	Rating	2.12	2.30	1.30
		q_m	1.47	1.36	2.97
		σ_S	0.80	1.14	2.05
	5	Rating	1.84	2.64	1.46
		q_m	1.62	1.10	2.05
		σ_S	0.79	0.63	1.27

where rating, maximum uncertainty parameter q_m and standard deviation σ_S have been calculated using the mean values obtained from the 50 closed loop polytopic systems.

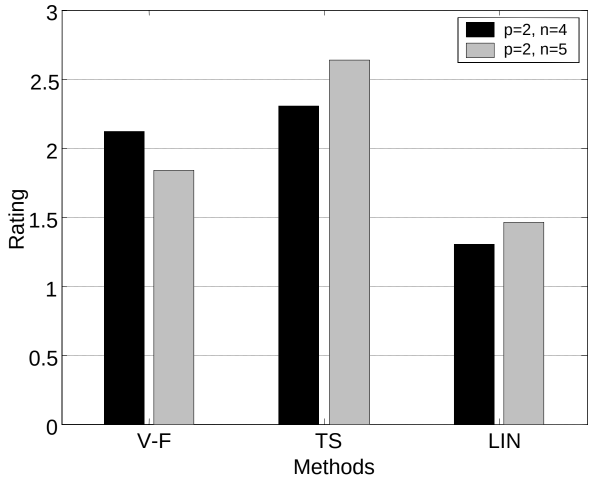

Fig. 3 – Result of robust stability evaluation in terms of rating

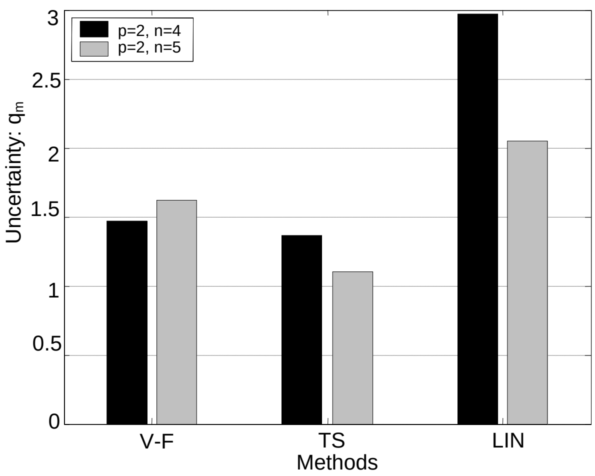

Fig. 4 – Result of robust stability evaluation in terms of uncertainty q_m

From all three quadratic stability methods (V-F, TS and LIN) the linearization method is the less conservative for both pairs ($n = 4$, $p = 2$) and ($n = 5$, $p = 2$): LIN method has obtained a minimum of points. The second place goes to the V-F method and the most conservative is the two-step method. When the affine system has become more complex (increased n) the LIN and TS methods have become more conservative on the contrary to the V-F method.

5 CONCLUSIONS

The paper provides a numerical comparison of three quadratic stability methods based on LMI conditions and a single Lyapunov function for continuous-time linear uncertain system with polytopic uncertainties. According to the proposed comparative test the linearization method provides less conservative results, generally. The presented algorithms are heuristic and may fail to determine the feedback gain, even if it exists. One of possible reasons could be too high requirements on performance. Advantage of static output feedback robust controller design by quadratic stability methods is that they are computationally simple.

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