ROBUST CONTROLLER DESIGN FOR MULTILINEAR INTERVAL UNCERTAINTY

GRMAN Ľ., HYPIUSOVÁ M., VESELÝ V.
Department of Automatic Control Systems,
Faculty of Electrical Engineering and Information Technology,
Slovak University of Technology
Ilkovičova 3, 812 19 Bratislava, Slovak Republic
e-mail: grman@kasr.elf.stuba.sk
hypiusov@kasr.elf.stuba.sk
vesely@kasr.elf.stuba.sk

Abstract: This paper deals with the robust controller design of linear SISO systems, with an uncertainty in multilinear interval form. In the first part of this work extremal transfer function of uncertainty system is derived, which with sufficient condition defines stability conditions for a general controller. The controller design, which guarantees the required phase margin, is carried out in the frequency domain. The proposed approach has been verified on a design of robust controller with variable time delay. This example illustrates application possibilities of extremal transfer functions.

Key words: robust controller, multilinear uncertainty, interval system and time delay.

1 INTRODUCTION

One of the elementary conditions for the behaviour of process control systems is to ensure control loop stability. Ensuring stability and performance is tightly connected with the controller design, which requires sufficiently accurate knowledge of plant model available, e.g. knowledge of the plant transfer function etc. In practice, however, object parameters often change, e.g. due to working point or controlled process structure changes, etc. Systems which parameters are changing within given intervals are referred to as interval systems. There are situations when several linear interval systems are connected in series. In such a case the global control object is considered to belong to the multilinear systems class. The task of the robust controller is to ensure closed-loop stability for all possible combinations of controlled system parameters from given intervals.

Recent developments in the robust control of systems with parametric uncertainty have been inspired by the Kharitonov theorem [5]. By means of this theorem it is sufficient to determine stability only of four Kharitonov polynomials. The next important substantial progress in the robust analysis of parameter stability have been developed in [1] with the stability proof for all groups of polynomials which coefficients vary within the arbitrary polytope. There are many robust controller design methods known from the literature - in the time domain [8] as well as in the frequency domain [3, 4, 6, 7].

In this paper we deal with robust stability of multilinear polytopic SISO systems, i.e. systems which uncertain interval parameters appear multilinearly in the transfer function. The controller is considered to be described by a transfer function with a fixed structure.

2 PRELIMINARIES AND PROBLEM FORMULATION

Consider a multilinear characteristic polynomial

$$p(s,q) = h_0(q) + h_1(q)s + h_2(q)s^2 + \cdots + h_n(q)s^n \tag{1}$$

where $q^T = [q_1 \dots q_p]$ is the vector of uncertain parameters
The vector q is from an uncertainty box given as

$$Q = \left\{ q : \ \underline{q}_i \le q_i \le \overline{q}_i \, , \ i = 1, 2, \cdots, p \right\} \tag{2}$$

where $h_i(q)$, $i = 0, 1, 2, \cdots, n$ are multilinear functions of entries of the vector q.
The family of multilinear characteristic polynomials is denoted by

$$p(s,q) = \{ p(s,q) : \ q \in Q \}. \tag{3}$$

Denote the vertices of Q and corresponding characteristic polynomials as follows

$$p_v(s,q) = \left\{ p(s,q) : \ q_i = \underline{q}_i \ \ or \ q_i = \overline{q}_i , \ i = 1, 2, \cdots, p \right\} = \{ v_1(s), \cdots, v_k(s) \} \tag{4}$$

where $k = 2^p$.

Let $\Delta(s)$ denote the convex hull of the vertex polynomials $p_v(s,q)$

$$\Delta(s) = \sum_{i=1}^{k} \lambda_i v_i(s) \qquad \qquad \sum_{i=1}^{k} \lambda_i = 1 \tag{5}$$

where $\lambda_i \in \langle 0, 1 \rangle$, $i = 1, 2, \cdots, k$.

Under the assumptions that

a) for any $q \in Q$ the degree of (1) is equal to n and
b) there exists q^* such that $p(s, q^*)$ is Hurwitz stable,

the family of characteristic polynomials is stable [2] if the following set of segments are stable

$$E(s) = \{ \lambda v_i(s) + (1 - \lambda) v_j(s) : \ v_i(s), v_j(s) \in p_v(s,q) \} \tag{6}$$

and $\lambda \in \langle 0, 1 \rangle$.

Note that (6) will not be stable if Q is not an axis–parallel box and the dependency on the uncertain parameters is polynomial rather than multilinear.

Consider the multilinear plant transfer function

$$M(s) = \frac{B(s, q_1, \cdots q_p)}{A(s, q_1, \cdots q_p)} \tag{7}$$

and a controller

$$R(s) = \frac{C(s)}{D(s)} \tag{8}$$

The problem to be solved can by formulated as follows:
Design the structure and parameters of controller (8) in such a way that the closed loop stability of the system described by the multilinear characteristic polynomial

$$P(s,q)=C(s)B(s,q)+D(s)A(s,q) \tag{9}$$

is guaranteed for any q from the uncertainty box Q (2).

3 MAIN RESULTS

The closed loop stability of the system (9) is guaranteed if the set of segments (6) are stable. Let us denote the vertex characteristic polynomials of (9) according to (4). For the element v_i of (4) we obtain

$$v_i = C(s)v_{Bi} + D(s)v_{Ai} \qquad i=1,2,\cdots,k \tag{10}$$

where v_{Bi} , v_{Ai} , $i=1,2,\ldots,k$ are the vertex polynomials of polynomial $B(s,q)$, $A(s,q)$, respectively.

The sets of multilinear characteristic polynomial segments are given by (6) and for E_i entries we obtain

$$E_i(s)=\lambda(C(s)v_{Bi}+D(s)v_{Ai})+(1-\lambda)(C(s)v_{Bj}+D(s)v_{Aj})$$

or after some simple manipulation

$$E_i(s)=C(s)\lfloor\lambda v_{Bi}+(1-\lambda)v_{Bj}\rfloor+D(s)\lfloor\lambda v_{Ai}+(1-\lambda)v_{Aj}\rfloor \tag{11}$$

For all entries of $E(s)$ are should rewrite the characteristic polynomial

$$p(s)=\frac{C(s)}{D(s)}\frac{\lambda v_{Bi}+(1-\lambda)v_{Bj}}{\lambda v_{Ai}+(1-\lambda)v_{Aj}}+1 \tag{12}$$

or the extremal transfer functions of multilinear plant (7) should read as follows

$$M_E(s)=\frac{\lambda v_{Bi}+(1-\lambda)v_{Bj}}{\lambda v_{Ai}+(1-\lambda)v_{Aj}}, \qquad i\neq j, \qquad \begin{array}{ll} i,j=1,2,\cdots,\dfrac{2^p!}{2(2^p-2)!} & \text{for multilinear case and} \\ i,j=p2^{p-1} & \text{for linear case} \end{array} \tag{13}$$

where $\lambda\in\langle 0,\, 1\rangle$.

The typical case of a multilinear plant (7) is the transfer function of multilinear polytopic system with variable time delay.

$$G(s)=\frac{(b_{01}(s)+q_{11}b_{11}(s)+\cdots)(b_{02}(s)+q_{12}b_{12}(s)+\cdots)\cdots}{(a_{01}(s)+q_{11}a_{11}(s)+\cdots)(a_{02}(s)+q_{12}a_{12}(s)+\cdots)\cdots}e^{-sD} \tag{14}$$

where $a_{ij}(s), b_{ij}(s)$ are known polynomials of given degree, $q_{ij}\in\langle \underline{q}_{ij},\, \overline{q}_{ij}\rangle$ are uncertainty parameters $i,j=1,2,\cdots,\ i\neq j$ and $D\in\langle \underline{D},\, \overline{D}\rangle$ varies within the given time delay interval.

One possible controller design procedure may consist of following steps:

1) Calculate the vertex polynomials of $B(s,q)$ and $A(s,q)$ denoted as v_{Bi}, v_{Ai}, $i = 1,2,\cdots,k$.
2) Calculate the set of extremal transfer functions (13).
3) Using the frequency domain approach determine the robust controller structure and parameters.

Remarks:

1) For linear interval systems with necessary and sufficient stability conditions (or polytopic systems) the number of segments (11) is equal to $p2^{p-1}$.
2) For multilinear systems with sufficient stability conditions (or multilinear polytopic systems) the number of segments (11) is equal to $\frac{k!}{(k-2)!\,2}$ where $k = 2^p$.

4 EXAMPLE

Consider the transfer function of boil heating process as follows

$$G(s) = \frac{(0.7s + 1 + q_{11}0.2s + q_{21}0.15s)(1.5s + 1 + q_{12}0.3s + q_{22}0.05s)}{(s^2 + 38s + 0.9 + q_{11}(5s + 0.3) + q_{21}0.12s)(s^2 + 113s + 1.1 + q_{12}5s + q_{22}3s)} e^{-sD},$$

where the time delay $D \in \langle 8; 13 \rangle$ and uncertainty parameters $q \in \langle -1; 1 \rangle$.

There are 5456 extremal transfer functions having been derived for multilinear interval uncertainty. The Bode diagram for extremal transfer functions $G_E(s)$ is shown in Fig. 1. In the frequency domain we have designed a robust PI controller in the form

$$R(s) = K\left(1 + \frac{1}{T_i s}\right) = \frac{100s + 1}{100s},$$

where the gain $K = 1$ and the integration time constant $T_i = 100$ [s].

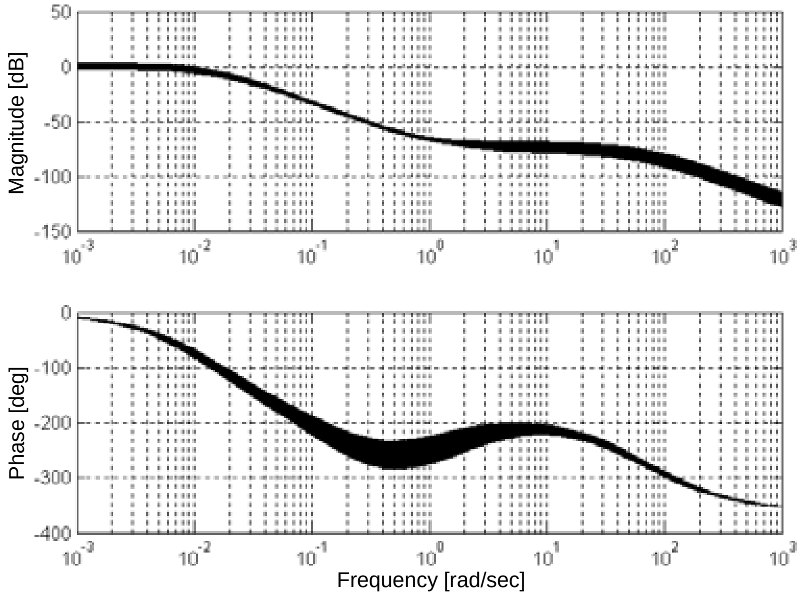

Fig. 1 – Bode diagrams of extremal transfer $G_E(s)$

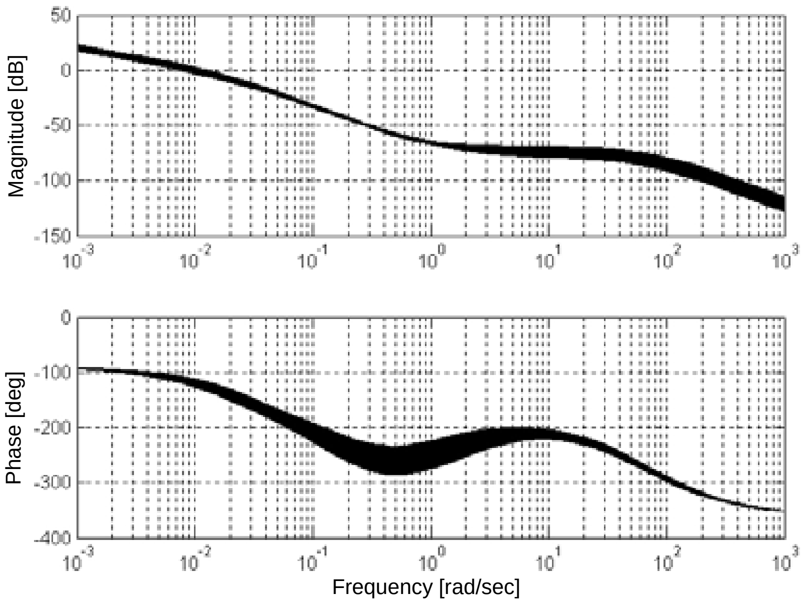

Fig. 2 – Bode diagrams of the open loop system: $G_E(s).R(s)$

Fig. 2 depicts the Bode diagrams of the open loop system. The achieved gain margin ΔK and phase margin $\Delta\varphi$ are:

$$\Delta K = 15.3\ \text{dB}$$
$$\Delta\varphi = 44.5°$$

From the Bode diagrams it is possible to see that the designed robust controller guarantees stability and performance for multilinear interval transfer function with uncertainties.

5 CONCLUSION

The main aim of this paper has been to present utilization of extremal transfer functions in the frequency domain robust controller design for multilinear interval transfer functions with variable time delay. The proposed robust controller design procedure with extremal transfer functions guarantee fulfillment of the necessary and sufficient stability conditions. The presented application of this approach to the boil heating process has resulted in a high robustness measure.

REFERENCES

[1] BARTLETT, A.C., HOLLOT, C.V., LIN, H.: Root Location of an Entire Polytope of Polynomials: It Suffices to Check the Edges. *Mathematics of Control, Signals and Systems*, vol. 1, 1988, pp. 61-71.

[2] BHATTACHARYYA, S.P., CHAPELLAT, H., KEEL, L. H.: Robust Control: The Parametric Approach, *Prentice Hall, Englewood Cliffs, N.J.*, 1995.

[3] HARSÁNYI, L., DÚBRAVSKÁ, M.: Robust Controller Design for Linear Systems with Parametric and Dynamic Uncertainties. *Journal of Electrical Engineering*, Vol. 52, No. 9-10, 2001, pp. 307-310.

[4] HARSÁNYI, L., DÚBRAVSKÁ, M.: Robust PID Controller Design. *Journal of Electrical Engineering*, Vol. 52, No. 1-2, 2001, pp. 19-23.

[5] KHARITONOV, V. L.: Asymptotic Stability of an Equilibrium Position of a Family of Systems of Linear Differential Equations. *Differential Equations,* vol. 14, 1979, pp. 1483-1485.

[6] KOZÁKOVÁ, A.: Robust Control of MIMO Systems in the Frequency Domain, *2nd IFAC Workshop*, NTDCS'97, 1997, pp. 415-420.

[7] TSYPKIN, Y., – POLYAK, B.: Frequency Domain Criteria for Robust Stability of Polytope of Polynomials, in Control of Uncertain Dynamic Systems. S. Bhattacharyya and L. Keel, ed., CRC Press, 1991, pp. 491-499.

[8] VESELÝ, V., KOZÁKOVÁ, A., PAPADOPOULOS, D. P.: Design of Robust Output Feedback Controller via LMI Approach. *Journal of Electrical Engineering*, Vol. 52, No. 9-10, 2001, pp. 273-277.

Acknowledgment

The work has been partially supported by the Slovak Scientific Grant Agency, Grant No. 1/7608/20.