ROBUST CONTROLLER DESIGN

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Abstract

This paper deals with the robust controller design for linear SISO systems, with uncertainty. In the first part of this work extremal transfer functions of linear and multilinear systems in polytopic or interval form are described. The controller design is carried out using two methods. The first design method uses the integral criterion I_{SE} (integral of the square of the error) and I_{TSE} (integral of the square of error multiplied by time). The second design method is carried out in the frequency domain and is based on the required phase margin and settling time. The example illustrates application possibilities of extremal transfer functions.

1 INTRODUCTION

One of elementary conditions for the behaviour of process control systems is to ensure control loop stability. Ensuring stability and performance is tightly connected with the controller design, which requires sufficiently accurate knowledge of plant model available, e.g. knowledge of the plant transfer function etc. The aim of identification of control system is not only finding an object model but also to determine range of parameter changes of the object to be controlled. In practice, however, object parameters often change, e.g. due to working point or controlled process structure changes, etc. A mathematical description of such an object then consists of an infinite number of transfer functions. In such case the controller design by extremal transfer functions can be carried out. The controller parameters are to be chosen so as to simultaneously stabilize a finite number of transfer functions representing the so-called extremal transfer function of object.

2 LINEAR MODELS OF PROCESS

Let us assume that a transfer function is a result of an experimental object identification. The parameters of transfer function may change in different manners and we can to include them to several classes [4].

2.1 Interval systems

In linear interval systems the coefficients of transfer function are changing independently within given intervals.

The interval transfer function of process is of the form:

$$G(s)=\frac{B(s)}{A(s)}=\frac{b_0+b_1s+b_2s^2+...+b_ms^m}{a_0+a_1s+a_2s^2+...+a_{n-1}s^{n-1}+s^n} \qquad (1)$$

where $b_i \in \left\langle \underline{b}_i, \overline{b}_i \right\rangle$, $i=0,...,m$, $a_j \in \left\langle \underline{a}_j, \overline{a}_j \right\rangle$, $j=0,...,n-1$

The number of transfer functions of the object is infinite as results from (1) and therefore a design of controller parameters cannot satisfy stability and performance requirements for the object (1) so that only several transfer functions from whole set can be stabilized.

We have carried out design of controller parameters for the interval system (1) using extremal transfer functions. The extremal transfer functions represent a finite number of object´s transfer functions with invariable parameters which uniquely determine stability of the whole set of transfer functions with the proposed controller.

We know several extremal transfer functions. In our case the extremal transfer functions [1] have been used

$$G_E(s)=\frac{K_B^i(s)}{\lambda K_A^j(s)+(1-\lambda)K_A^k(s)} \cup \frac{\lambda K_B^j(s)+(1-\lambda)K_B^k(s)}{K_A^i(s)} \qquad (2)$$

where $i \in (1,2,3,4)$; $(j,k) \in \{(1,2),(1,3),(2,4),(3,4)\}$;

$\lambda \in [0, 1]$, $K_B(s)$, $K_A(s)$ are Kharitonov polynomials.

The number of extremal transfer functions is independent from the order of the object (1) but depends on the step λ.

2.2 Polytopic systems

If during the identification it is found out that in the transfer function of the object some coefficients are changing simultaneously, then the transfer function of such an object can be written as follows

$$G(s)=\frac{B(s)}{A(s)}=\frac{b_0(s)+\sum_{i=1}^{p} q_i b_i(s)}{a_0(s)+\sum_{i=1}^{p} q_i a_i(s)} \qquad (3)$$

where $b_0(s), b_i(s), a_0(s), a_i(s)$, $i=1,2,...,p$ are known polynomials with fixed coefficients and coefficients q_i are unknown, but varying within a prescribed interval $q_i \in \left\langle \underline{q}_i, \overline{q}_i \right\rangle$. If we let to change coefficients $q_i = \underline{q}_i$ or $q_i = \overline{q}_i$ so we obtain 2^p transfer functions with fixed coefficients.

These transfer functions can be placed into vertices of a p dimensional uncertainty box and then the transfer function (3) describes a polytopic system.

Stability of the closed-loop with the transfer function (3) requires that the designed controller guarantees closed-loop stability with 2^p transfer functions of the object as well as closed-loop stability on each edge of the uncertainty box.

2.3 Multilinear systems

There are situations when several linear interval systems (1) or (3) are connected in series. In such a case the global controlled object is considered to belong to the multilinear systems class.

$$G(s)=\frac{B_1(s)B_2(s)\ldots B_p(s)}{A_1(s)A_2(s)\ldots A_p(s)} \tag{4}$$

where $\dfrac{B_i(s)}{A_i(s)}$, $i=1,2,...,p$ is linear interval or polytopic system.

We can obtain a multilinear system also in case of a controlled system with a variable time delay

$$G(s)=\frac{B(s)}{A(s)}e^{-Ds} \tag{5}$$

where D varies within the given time delay interval.

When we replace the term e^{-Ds} by a second-order Padé approximation

$$e^{-Ds}\approx\left(1-\frac{D}{2}s+\frac{D^2}{12}s^2\right).\left(1+\frac{D}{2}s+\frac{D^2}{12}s^2\right)^{-1} \tag{6}$$

From (5) and (6) it can be seen that we have obtained a multilinear system.

In the controller design we have used the extremal transfer function of a multilinear polytopic object

$$M_E(s)=\frac{\lambda v_{Bi}+(1-\lambda)v_{Bj}}{\lambda v_{Ai}+(1-\lambda)v_{Aj}}\ ,\ i,j=1,2,\cdots,\frac{2^p!}{2(2^p-2)!} \tag{7}$$

where $i\neq j$, v_B, v_A are vertex polynomials of numerator and denominator (4), p is the number of uncertainties considered, $\lambda\in\langle 0,1\rangle$.

3 ROBUST CONTROLLER DESIGN

The robust controller design will be carried out by two methods.

3.1 First design method

The controller design is carried out based on the required settling time t_{reg}, phase margin $\Delta\varphi_0$ and maximum overshoot η_{max}. The method for determining η_{max} and t_{reg} from the open loop Bode diagrams was designed by *Reinisch* [3].

Reinisch derived with a sufficient precision for all systems, whose open loop transfer function include integrator and is of the form

$$G_O(s)=\frac{K}{s}\frac{\prod_{j=1}^{m}(1+\tau_j s)}{\prod_{i=1}^{n-1}(1+\tau_i s)}\ ,\quad m\leq n \tag{8}$$

the following performance measures:

Dependency of the maximum overshoot η_{max} on the argument of the open loop frequency transfer function $\varphi(\omega_0)$ is

$$\eta_{\max}=e^{-\frac{\pi}{\sqrt{-\frac{4\cos\varphi(\omega_0)}{\sin^2\varphi(\omega_0)}-1}}}.100\ ,\quad \Delta\varphi_0=180+\varphi(\omega_0) \tag{9}$$

If we consider a dead zone δ = 5 [%] and a change of damping coefficient $b\in$ (0.25, 0.65) then

$$\frac{\pi}{\omega_0}<t_{reg}<4\frac{\pi}{\omega_0} \tag{10}$$

The inequality (10) specifies the settling time with a sufficient precision from the crossover frequency ω_0 at which the $G_0(j\omega)$ magnitude is equal 1.

Based on the required phase margin and using (9) and (10) we can design a robust controller from Bode diagrams.

3.2 Second design method

The controller design is carried out respectively by the criterion of the minimum of integral of the error square (I_{SE}) and the criterion of minimum integral of error square multiplied by time (I_{TSE}). The integral performance criterions provide information of the control process on the basis of integral error for all time values.

The algorithm for calculating I_{SE} designed by Nekolny [2] comes from the Parseval's integral in form

$$I_{SE}=\int_0^\infty [e(t)]^2dt=\frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty}E(s)E(-s)ds \tag{11}$$

where *E(s)* is the Laplace transform of the tracking error

$$E(s)=\frac{D(s)}{A(s)}=\frac{d_{n-1}s^{n-1}+\cdots+d_1s+d_0}{a_ns^n+\cdots+a_1s+a_0} \tag{12}$$

The integral of the square of error multiplied by time can be written as

$$I_{TSE}=\int_0^\infty te^2(t)dt \tag{13}$$

For the integral (13) formulas in the closed form have been derived.

Design of unknown controller parameters have been carried out by *minimax problem*

$$\min_x \max_{\{F_i\}}\{F_i(x)\} \tag{14}$$

which is realized by *fminimax* function in the Matlab Optimization Toolbox. This function minimizes the worst-case value of a set of multivariable functions.

4 EXAMPLES

Example 1:

Consider the transfer function of a glass furnace with output of temperature to supplied energy in form of a multilinear polytopic system with variable time delay as follows:

$$G(s)=\frac{b_0(s)+q_1 b_1(s)}{a_0(s)+q_1 a_1(s)}e^{-sD} \tag{15}$$

where $b_0(s)=3s+0.2$; $b_1(s)=0.5s+0.05$; $q\in\langle -1;\, 1\rangle$;

$a_0(s)=5515s^3+2925s^2+120s+1$; $D\in\langle 5;\, 10\rangle$;

$a_1(s)=495s^3+175s^2+10s$.

The time delay e^{-sD} has been replaced by a second-order Padé approximation (6).

There are 36 extremal transfer functions (7) derived for multilinear polytopic uncertainty and to each of them a Laplace transform of the tracking error is assigned (12). The robust PID controller designed by Nekolny algorithm for calculating I_{SE} and by *minimax* problem is of the form

$$R(s)=P\left(1+\frac{1}{T_I s}+T_D s\right) \tag{16}$$

with $P = 45.8$, $T_I = 112.7$ [*s*], $T_D = 11.3$ [*s*].

The closed loop step responses of output value (*y*) and control value (*u*) under the designed robust controller are shown in Fig. 1.

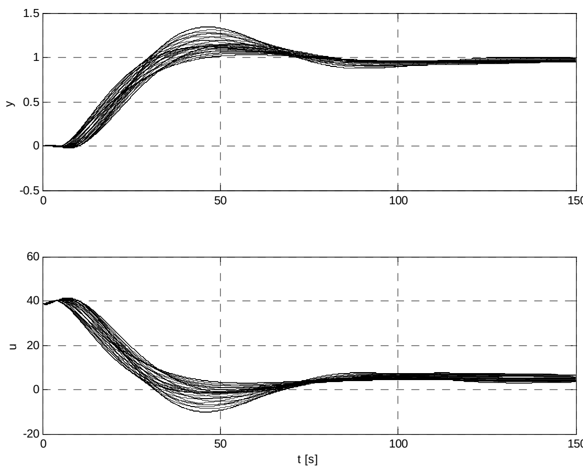

Fig. 1: Step responses of the closed loop system

The step responses (Fig. 1) of output value (*y*) have a maximum overshoot η_{max} = 34.4 [%] and a settling time t_{reg} = 130 [*s*].

Example 2:

Consider the process from Example 1 without the variable time delay and described by the interval transfer function as follows:

$$G(s)=\frac{b_1 s+b_0}{a_3 s^3+a_2 s^2+a_1 s+a_0}, \tag{17}$$

where b_1 = [2.5, 3.5], b_0 = [0.15, 0.25], a_3 = [5020, 6010], a_2 = [2750, 3100], a_1 = [110, 130], a_0 = 1.

There are 192 extremal transfer functions derived for linear interval uncertainty.

Parameters of the robust PID controller designed on the basis of the guaranteed phase margin $\Delta\varphi_0$ = 75° and settling time t_{reg} = 90 [*s*] are *P* = 61, $T_I = 114.8$ [*s*], $T_D = 0.24$ [*s*].

The closed loop step responses of output value (*y*) and control value (*u*) under the designed robust controller are shown in Fig. 2.

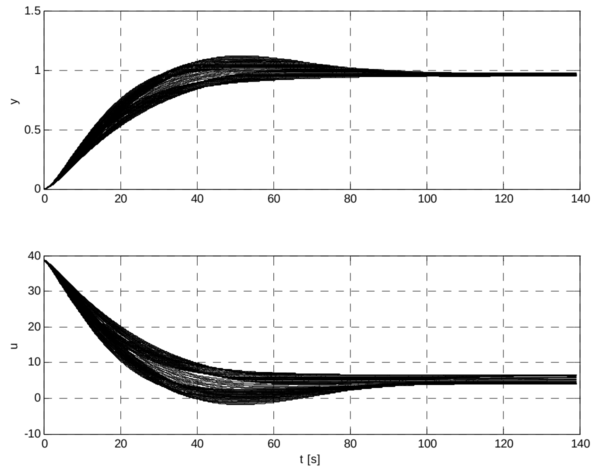

Fig. 2: Step responses of the close loop system

The step responses (Fig. 2) of output value (*y*) have a maximum overshoot η_{max} = 12 [%] and t_{reg} = 110 [*s*].

5 CONCLUSIONS

The aim of this paper was to present chosen real models of objects for which there are robust controller design methods and software support available.

Advantage of the design by the criterion of the minimum of integral of the error square (I_{SE}) is a general applicability but its drawback is over-estimation of big and under-estimation of small tracking errors. Therefore the response of the designed control system is more oscillating with a longer settling time.

Main benefits of the by Reinisch derived basic performance measures are their simple calculation and a sufficiently general validity.

6 REFERENCES

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